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A Logical Approach to Argumentation
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Introduction. Any result of a discursive activity can be analyzed from at least two distinct perspectives: a logical one (aiming to establish the correctness of the thoughts, and the way each thought is related with the next one, in a clear, easy to follow judgement); a semiotic perspective would offer the possibility to understand the way the utterer has used the linguistic material (as signs, in communication), in order to reach the goal s/he is after [1], taking into account the relations between the signs and their users, as well as the interaction between the participants to the argument. In the present paper, we intend to stress out the logical perspective.

1. The structure of argumentation. The argumentation is one of the most widely spread dialogic relations. By using it, individuals submit to critical examination pieces of information in the form of statements connected by a rational relation of inference, meaning that some of them serve as logical backing for the other ones. The argumentation has a well-determined purpose: solving the conflicts of opinion through negotiation [2]. When there is a conflict of opinion between two individuals (a discrepancy of the views on supporting or rejecting a thesis), no other rational way of resolving it is easier than furnishing the proves (evidence, arguments, reasons) crediting or discrediting that thesis. According to the backing force of the proves, the conflict of opinion will be solved in favor of one person or the other of those involved in the dialogic relation.

Even from this brief presentation, we realize that argumentation is, in its most elementary form, a relation between a given fact (the thesis of argumentation) and something that backs up this given fact (the backing or reason of argumentation):

\[ T \text{(thesis of argumentation)} \leftarrow R \text{(reason of argumentation)} \]

a relation that constitutes the central element of the “Toulmin model” of investigation of argumentation [3], which is analyzed in detail in the exegeses on the matter [4]. We can define
argumentation as an organization of statements through the acts of reasoning aiming to support another statement as true or false and to persuade the interlocutor of its truthfulness or falseness.

We have here the difference between argumentation and reasoning. In the act of reasoning, premises are given, and what we need to determine is the conclusion that necessarily results from them (if the reasoning is valid), whereas in the act of argumentation the thesis (conclusion) is given, and we have to look for the proves (arguments) that found it. In reasoning, the mind works backwards, from the premises to the conclusion. In argumentation, it works progressively, from the given thesis to the foundations (reasons) supporting it.

2. Argumentative functions and forms. An argumentation can only have one thesis (or claim), but it can consist of more than one reason (elementary statements used to prove the thesis). In a critical dispute we can argue in favor of many theses, but then, naturally, we have multiple argumentations. We shall now focus on elementary forms of argumentation, those in which the thesis (T) is an elementary statement (which cannot be divided into other statements), and the reason (R) is also an elementary statement.

As stated before, the argumentation is a relation of logical support from the reason (R) to the thesis (T). We shall call these two elements of the relation of argumentation argumentative functions [5]. The function T (read “to be the thesis …”) as well as the function R (read “to be the support …”) can only function properly in the practice of argumentation if they are assigned a statement satisfying the requirements of a logical relation of rational founding. We shall call these elementary statements assigned to the argumentative functions (T) and (R) and which satisfy the relation of rational founding arguments of the argumentative functions.

An elementary argumentation is hence made of two argumentative functions (T and R) and of their respective arguments (which we notate by p and q):

\[ T(p), R(q) \]

which can be read: “the statement q is a rational foundation for the thesis actualized in the statement p”. It is like in this example, from Balzac:

“Moral sufferings (…) provoke less pity, because we cannot see them”
which is an actualization of the given formula that can be read – in terms of argumentative relation – as follows: “the statement «we cannot see the moral sufferings» is a rational foundation for the support of the thesis actualized in the statement «Moral sufferings provoke less pity»”.

To lay down the outline of the construction in this field, we need two elementary logical operations: stating and denying (supporting or being against). We find them essential for the logical analysis of argumentation, too. So, we can apply the operation of stating and the operation of being against to the two argumentative functions and to their arguments, as shown in the formula:

$$\pm T(\pm p), \pm R(\pm q)$$

Every instantiation of this formula generates a certain combination between the argumentative functions, the arguments of each of the functions, and the operations of stating or being against applied to the functions and to the arguments. We shall call these combinatorial instantiations argumentative forms. Without explicitly taking support in the given formula, Apothéloz clearly presents the 16 possible argumentative forms of an elementary argumentation [5: 70-72]:

$$f_1 : T(p), R(q) \quad q \text{ is a reason for a thesis of the form } p$$
$$f_2 : T(p), R(-q) \quad -q \text{ is a reason for a thesis of the form } p$$
$$f_3 : T(-p), R(q) \quad q \text{ is a reason for a thesis of the form } -p$$
$$f_4 : T(-p), R(-q) \quad -q \text{ is a reason for a thesis of the form } -p$$
$$f_5 : T(p), -R(q) \quad q \text{ is not a reason for a thesis of the form } p$$
$$f_6 : T(p), -R(-q) \quad -q \text{ is not a reason for a thesis of the form } p$$
$$f_7 : T(-p), -R(q) \quad q \text{ is not a reason for a thesis of the form } -p$$
$$f_8 : T(-p), -R(-q) \quad -q \text{ is not a reason for a thesis of the form } -p$$
$$f_9 : -T(p), R(q) \quad q \text{ is a reason against a thesis of the form } p$$
$$f_{10} : -T(p), R(-q) \quad -q \text{ is a reason against a thesis of the form } p$$
$$f_{11} : -T(-p), R(q) \quad q \text{ is a reason against a thesis of the form } -p$$
$$f_{12} : -T(-p), R(-q) \quad -q \text{ is a reason against a thesis of the form } -p$$
Each one of these argumentative forms is the expression of a practical way of argumenting, and it can be easily detected in every day argumentations.

3. Logical relations between argumentative forms. The starting point of the analysis of the logical relations between the argumentative forms is made of the concept of argumentative operator, and of its interpretation by the means of the interstatements connectives [6]. Even by having just a look over the 16 argumentative forms above, we can see that they fall in groups of four by the argumentative operator, which can be:

\[ C_1: \text{“...is a reason for...”} \quad \text{(formulae } f_1 - f_4) \]
\[ C_2: \text{“...is not a reason for...”} \quad \text{(formulae } f_5 - f_8) \]
\[ C_3: \text{“...is a reason against...”} \quad \text{(formulae } f_9 - f_{12}) \]
\[ C_4: \text{“...is not a reason against...”} \quad \text{(formulae } f_{13} - f_{16}) \]

The argumentative operator \( C_1 \) (“...is a reason for...”) is the expression of the support a given argument brings to a thesis. This is why we shall call it argumentative operator of support (S). The argumentative operator \( C_3 \) (“...is a reason against...”) serves to refutate a thesis by the means of a given argument. It will be called argumentative operator of refutation (N). The argumentative operator \( C_2 \) (“...is not a reason for...”) is the negative of the former, and this is why it will be called argumentative operator of counter support (CS), whereas the argumentative operator \( C_4 \) (“...is not a reason for denying...”) is the negation of the latter in the paragraph before, so it will be called argumentative operator of counter refutation (CN).

The argumentative operators we have analyzed connect two elementary sentences: the reason-sentence \( q \) and the thesis-sentence \( p \). We can see that \( C_1 \) and \( C_2 \) are contradictory to one another (the argumentative forms constructed using these operators, where the functions and
the arguments are identical, are contradictory to one another). The argumentative operators $C_1$ and $C_3$ are in a relation of sub contrary opposition (an argumentative form cannot not be either a reason for nor a reason against; otherwise it wouldn’t be an argumentative form!).

Finally, the argumentative operators $C_4$ and $C_2$ are in a relation of contrary opposition (a given argumentative form cannot fall at the same time in the class “…is not a reason for…” and in the class “…is not a reason against…”). We can easily see that the argumentative operators we have analyzed “fit” a well-known logical structure, the logical square of oppositions. A structure that emphasizes on the logical relations of: contrariety (between $C_4$ and $C_2$), subcontrariety (between $C_1$ and $C_3$), contradiction (between $C_4$ and $C_3$, and between $C_1$ and $C_2$), subalternation (between $C_4$ and $C_1$, and between $C_2$ and $C_3$), and supralternation (between $C_1$ and $C_4$, and between $C_3$ and $C_2$), of the constructed argumentative forms with the correspondent argumentative operators:

$C_4$ (“…is not a reason against…”) \hspace{2cm} C_2 (“…is not a reason for…”) \hspace{2cm} C_1 (“…is a reason for…”) \hspace{2cm} C_3 (“…is a reason against…”)

Paying attention to the alternate presence of the signs (+, -) of the argumentative functions and of their respective arguments, we can construct four structures of logical square each of them having four argumentative forms obeying to the exigencies of the logical relations of such a structure. Here is one of the four possible instantiations (structure 1):

$-T(p), -R(q) \hspace{2cm} T(p), -R(q) \hspace{2cm} T(p), R(q) \hspace{2cm} -T(p), R(q)$

The three others can be easily determined.
4. Logical connectives and the interpretation of the argumentative operators. How could we express the four argumentative operators by the means of intersentential connectives? Saying that a given sentence (q) is a reason for a given thesis (p) means that the truthfulness of the former is the sufficient condition for supporting the truthfulness of the latter. Therefore, the reason-sentence is the sufficient condition. Knowing that the truth of a given thesis couldn’t be proved means to know that there hasn’t been any reason-sentence. Thus, the thesis-sentence is the necessary consequence. Then, the argumentative operator C₁ (“…is a reason for…”) is the expression of the relation of sufficient-necessary conditioning existing between the reason-sentence and the thesis-sentence. The relation of sufficient-necessary conditioning between two sentences is expressed in modern logic by the means of the logical connective called material implication (“⇒”) [7]. And there will be four argumentative forms expressed by the means of implication:

\[
\begin{align*}
  f₁ : & & R(q) \Rightarrow T(p) \\
  f₂ : & & R(\neg q) \Rightarrow T(p) \\
  f₃ : & & R(q) \Rightarrow T(\neg p) \\
  f₄ : & & R(\neg q) \Rightarrow T(\neg p)
\end{align*}
\]

If the argumentative functor C₂ (“…is not a reason for…”) is the contrary of the one before it, expressing it by the means of sentential connectives will be possible with the help of an operator that “reverses” the truth matrix of the implication. This is the non-implication (“⊅”). Obviously, there will be four argumentative forms expressed by the means of non-implication, and it is easy to find them,

The argumentative operator C₃ (“…is a reason against…”) means that the presence of the reason-sentence makes impossible the stating of the thesis-sentence, and that the support for the thesis-sentence cannot be secured using the reason-sentence. Such an argumentative operator points out the fact that in some acts of argumentation the two sentences (the reason and the thesis) cannot coexist, the presence (the stating) of one of them indicating the absence (the denying) of the other. These sentences are in a relation of contrary opposition that is reproduced by the logical
connective called \textit{incompatibility} \( (\uparrow) \). Among the 16 argumentative forms there are four that can be expressed by the means of the incompatibility. As the argumentative operator \( C_4 \) \( (\ldots \text{is not a reason against...}) \) is \textit{the contradictory negation} of the one before, the logical connective that will express it is the negation of the incompatibility, that is the operator \textit{conjunction} \( (\&) \). And there will be four argumentative forms that will be expressed using conjunction.

This is how the four argumentative operators are expressed:

\begin{itemize}
    \item \( C_1 \): "\ldots\text{is a reason for...}" = \( \supset \)
    \item \( C_2 \): "\ldots\text{is not a reason for...}) = \( \supsetp \)
    \item \( C_3 \): "\ldots\text{is a reason against...}" = \( \uparrow \)
    \item \( C_4 \): "\ldots\text{is not a reason against...}" = \( \& \)
\end{itemize}

And they fall in the structure of the logical square of oppositions like this [8]:

\begin{center}
\begin{tikzpicture}
    \node at (0,0) (p) {\&};
    \node at (1,0) (q) {\supset};
    \node at (1,-1) (r) {\supsetp};
    \node at (0,-1) (s) {\uparrow};
    \draw[->] (p) -- (q);
    \draw[->] (q) -- (r);
    \draw[->] (r) -- (s);
    \draw[->] (s) -- (p);
\end{tikzpicture}
\end{center}

If we put these argumentative forms in expression using sentential connectives to interpret the argumentative operators, then the structure 1 becomes (structure 2):

\begin{center}
\begin{tikzpicture}
    \node at (0,0) (p) {q\&p};
    \node at (1,0) (q) {q\supsetp};
    \node at (1,-1) (r) {q\supsetp};
    \node at (0,-1) (s) {q\uparrow};
    \draw[->] (p) -- (q);
    \draw[->] (q) -- (r);
    \draw[->] (r) -- (s);
    \draw[->] (s) -- (p);
\end{tikzpicture}
\end{center}

where we can use the logical methods we know to verify the logical relations within these argumentative forms. Let’s take as an example the relation of contrariety between the argumentative forms \( (q\&p) \) and \( (q\supsetp) \):
Using the method of the truth-values tables [9], it shows a valid formula in propositional logic, and hence a logical relation of incompatibility between the two argumentative forms.

5. Direct and indirect argumentative forms. By the nature of the argumentative operator they illustrate, the 16 argumentative forms fall into four classes, each of them playing a well-determined part: the class of statements (argumentative forms ranging from $f_1$ to $f_4$), the class of refutations (argumentative forms ranging from $f_9$ to $f_{12}$), the class of counterstatements (the argumentative forms ranging from $f_5$ to $f_8$), and the class of counter refutations (the argumentative forms ranging from $f_{13}$ to $f_{16}$). We call these forms direct argumentative forms, as the reason-sentence ($q$) supports or denies the thesis-sentence ($p$) without relying on any intermediary deduction, thus without taking supplementary steps.

But, there is also the possibility to determine indirect argumentative forms, where the support for the thesis is ensured by the means of a deduction process that valorizes the logical relations between the direct argumentative forms, mainly those logical relations that are emphasized by the structure of the logical square of oppositions.

Let us begin with the relation of implication (sufficient-necessary conditioning, sub- alternation) between the argumentative forms ($q\&p$) and ($q\supset p$), as shown in the structure 2.

Taking into account this relation, we shall have:

\[(q\&p) \supset (q\supset p)\]

If we can somehow determine that there is a relation of conjunction between $q$ and $p$, then we can reach the conclusion that:

\[[(q\&p) \supset (q\supset p)]\&(q\&p)\]
We have here an elementary inference from Logic (the Ponendo-Ponens modus) that allows supporting as true the argumentative form \((q \supset p)\). The full argumentative process is:

\[
\{ [(q \& p) \supset (q \supset p)] \& (q \& p) \} \supset (q \supset p)
\]

As the conclusion of a valid reasoning, the argumentative form \((q \supset p)\) is true. Which means that the reason-sentence \((q)\) supports the thesis-sentence \((p)\).

This could not be proved directly, and two helping elements where necessary: the fact that we knew the implication from \((q \& p)\) to \((q \supset p)\), and the fact that \((q \& p)\) is a true argumentative form. With this help we have created a valid reasoning (Ponendo-Ponens) able to back up the fact that the argumentative form \((q \supset p)\) is also true. This is where the indirect backing comes from, and it generated the respective argumentative form.

Indirect argumentative forms can be constructed starting from each of the relations between the direct argumentative forms. Identifying them is easy for those who can use the virtues of the structure logical square of oppositions. Even though these determinations can look like theoretical constructions with no relevance in the praxis of argumentation, we fight this prejudice and state that these argumentative forms (as well as their combinations) are permanently present in every day disputes.

Here is a fragment from Espinoza’s Ethics:

“My justification is this: there is nothing that happens in the nature and which the nature can be charged with as a vice, because the nature is always the same.”

This discursive sequence could be structured, as an argumentative forms, as follows:

\[
\text{T} \quad \text{R}
\]

“…there is nothing that happens \((because)\) “…the nature is always the same” in the nature which the nature could be charged with as a vice”

But this is an indirect backing, which, in fact, is of the form:

\((p_1)\) If the nature is always the same, then nothing happens in the nature that could be held against it as a vice
(p₂) But the nature is always the same

c ) Then: In the nature nothing happens that could be held against it as a vice.

And this is a practical expression of the indirect argumentative form we have just analyzed.

6. Conclusions. The present attempt of formal analysis of argumentation emphasizes important aspects of the theory of argumentation, and of the practice of argumentation:

(a) The diversity of the ways we take in critical disputes with our fellow creatures, as confusing and discouraging as it may seem to those trying to reveal the “order of argumentation”, is, in our view, the expression of combinations and relatedness of basic argumentative forms, which can be maintained under cognitive control by an attentive analysis of the relations of backing, which occur in the practice of argumentation. Our analysis brings to attention the ideal of argumentation we all are seeking, but we think that the practice of argumentation cannot be significantly far from this ideal without risking the danger of sophisms. The formal analysis tried to make some order at the level of the basic argumentative forms.

(b) This investigation leads also to another conclusion: we have at our disposal many possibilities to support or to refute a thesis that is brought under critical debate. If we only think of an infrequent fact in the practice of argumentation, which is that a thesis is supported by just one reason-sentence, we have 16 direct argumentative forms at hand, plus the indirect ones – which can rise to more than 70, probably –, so we can realize the diversity of the possibilities we have. But what if we imagine a thesis supported by two, three, or more reason-sentences? A question now: why is it that argumentations vary from an individual to another? An answer, among other, could be: because each of the individuals taking part in the argumentation valorizes different argumentative forms or different combinations of argumentative forms. We can back up a thesis directly, we can back it up by assuming its counter refutation; we deny a thesis by supporting its contrary, and so on so forth.
Notes and References


